Study Guide for Exam 2

1. You are supposed to know how to comopute the integration of the form

(1) $\int \sin^m x \cos^n x \; dx$

(2) $\int \tan^m x \sec^n x \; dx$

Example Problems

1.1. Compute the following integrals:

(i) $\int \sin^3 x \cos^3 x \; dx$

(ii) $\int \sin^4 x \cos^2 x \; dx$

(iii) $\int \sec^4 x \tan^2 x \; dx$

(iv) $\int \sec^3 \tan x \; dx$

(v) $\int \sec^3 x \; dx.$

1.2

(i) Compute $\int \sec x \; dx = \int \frac{1}{\cos^2 x} \cos x \; dx = \int \frac{1}{1-\sin^2 x} \cos x \; dx$ using the substitution $u = \sin x$ and then using the partial fraction.

(ii) Check that the result obtained in (i) coincides with the well-known formula $\int \sec x \; dx = \ln|\sec x + \tan x| + C$.

2. You are supposed to know how to use the 3 types of trigonometric substitution, and carry out the integration accordingly.

(1) $\sqrt{a^2 - x^2}, x = a\sin\theta, dx = a\cos\theta, \sqrt{a^2 - x^2} = a\cos\theta,$

(2) $\sqrt{a^2 + x^2}, x = a\tan\theta, dx = a\sec^2\theta, \sqrt{a^2 + x^2} = a\sec\theta,$

(3) $\sqrt{x^2 - a^2}, x = a\sec\theta, dx = a\tan\theta\sec\theta, \sqrt{x^2 - a^2} = a\tan\theta.$

Example Problems

2.1. Compute the following integrals:

(i) $\int \sqrt{7x^2 + 1} \; dx$

(ii) $\int \frac{x}{\sqrt{3 - 2x - x^2}} dx$

(iii) $\int \sqrt{5 - 4x^2} \; dx$

3. You are supposed to know
   ◦ the proper form of the partial fractions,
   ◦ how to determine the appropariate constants apperaring in the partial fraction,
   ◦ how to compute the integral accordingly.

Example Problems

3.1. Compute the following integrals:

$$\text{(i)} \quad \int \frac{x^2}{(x-1)^2}\,dx$$
$$\text{(ii)} \quad \int \frac{x+2}{x^2+2x+2}\,dx$$
$$\text{(iii)} \quad \int \frac{x}{(x+1)(x-1)(x-2)}\,dx$$
$$\text{(iv)} \quad \int \frac{x^2}{(x-1)^2(x^2+1)}\,dx$$

4. You are supposed to know how to compute the arc length by the formula

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy.$$

Example Problems

4.1. Problems 9 and 11 in Section 8.1 on Page 549 of the textbbok

4.2. Webassign HW 17 Problems 1,2,3,4.

5. You are supposed to know how to compute the area of the surface obtained

(1) by rotating the curve around the x-axis

$$S = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_c^d 2\pi y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

(2) or, by rotating the curve around the y-axis

$$S = \int_a^b 2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_c^d 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy.$$

Example Problems

5.1. Webassign HW 17 Problems 5, 7, 8.

5.2. Find the formula for the surface area of S where S is obtained by revolving the upper part of the ellipse $\frac{y^2}{2}+\frac{x^2}{4}=1$ about the x-axis.

6. You are supposed to be able to approximate the value of the integration using the following 3 methods

- Midpoint rule,
- Trapezoidal rule,
- Simpson's rule.

Example Problems

6.1. Give the estimation of the integral $\int_0^1 x^{3/2}dx \approx T_4$ by applying the trapezoidal rule with $n = 4$.

6.2. We use Simpson's rule with $n = 4$ to approximate the value of

$$\int_1^2 x^2 dx \approx S_4.$$

What is the error

$$\left| \int_1^2 x^2 dx - S_4 \right|$$

between the true value and the approximation using Simpson's rule ?

7. You are supposed to know why a given improper integral is improper, and accordingly to be able to determine if the given improper integral is convergent/divergent. In case it is convergent, you should be able to compute its value.

Example Problems

7.1. Evaluate the following improper integrals

(i) $\displaystyle\int_0^\infty \frac{e^x}{e^{2x}+1}\,dx$

(ii) $\displaystyle\int_0^\infty \frac{e^{2x}}{e^{2x}+1}\,dx$

(iii) $\displaystyle\int_0^9 \frac{1}{x-1}\,dx$

(iv) $\displaystyle\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx$

(v) $\displaystyle\int_{-\infty}^\infty x\,dx$

(vi) $\displaystyle\int_{-\infty}^\infty xe^{-x^2}\,dx$

8. You are supposed to be able to compute the coordinates $(\overline{x}, \overline{y})$ of the centroid of a given fugure.

$$\begin{aligned} \overline{x} &= \frac{\displaystyle\int_a^b x\{f(x) - g(x)\}\, dx}{\displaystyle\int_a^b \{f(x) - g(x)\}\, dx}, \\ \overline{y} &= \frac{\displaystyle\int_a^b \frac{1}{2}\{f(x)^2 - g(x)^2\}\, dx}{\displaystyle\int_a^b \{f(x) - g(x)\}\, dx}. \end{aligned}$$

Example Problems

8.1. Webassign HW 18 Problems 2,3,4,5,6

8.2. Example 4 of Section 8.3 on Page 563 of the textbook

8.3. Find the centroid of the region D bounded by the curves $y = x^3$, $x + y = 2$, and $x = 0$. (Note that this one is different from the one in Webassign.)

9. You are supposed to be able to determine if a given sequence is convergent/divergent. In case it is convergent, you should be able to compute its limit.

Example Problems

9.1. Webassign HW 19 Problems 7,8,9,10, 11.